$$= \begin{vmatrix} c\alpha-d\beta+(c\beta+d\alpha)\sqrt{-1} & c\gamma-d\delta+(c\delta+d\gamma)\sqrt{-1} \\ -c\gamma+d\delta+(c\delta+d\gamma)\sqrt{-1} & c\alpha-d\beta-(c\beta+d\alpha)\sqrt{-1} \end{vmatrix}$$

$$+ \begin{vmatrix} a\alpha-b\beta+(a\beta+b\alpha)\sqrt{-1} & a\gamma-b\delta+(a\delta+b\gamma)\sqrt{-1} \\ -a\gamma+b\delta+(a\delta+b\gamma)\sqrt{-1} & a\alpha-b\beta-(a\beta+b\alpha)\sqrt{-1} \end{vmatrix}$$

or $(a^2+b^2+c^2+d^2)(\alpha^2+\beta^2+\gamma^2+\delta^2)=(c\alpha-d\beta)^2+(c\beta+d\alpha)^2$
$+(c\gamma-d\delta)^2+(c\delta+d\gamma)^2+(a\alpha-b\beta)^2+(a\beta+b\alpha)^2+(a\gamma-b\delta)^2+(a\delta+b\gamma)^2$.

Euler's Theorem is an easy corollary of this, and *vice-versa*.

University of Mississippi, March, 1896.

A METHOD OF SOLVING QUADRATIC EQUATIONS.

By Prof. HENRY HEATON, M. Sc., Atlantic, Iowa.

Let it be required to solve the equation

$ax^2+bx+c=0$..... ....(1).

Transposing the middle term we have

$ax^2+c=-bx$..... ....(2).

Squaring, $a^2x^4+2acx^2+c^2=b^2x^2$..............................(3).

Subtracting $4acx^2$, $a^2x^4-2acx^2=(b^2-4ac)x^2$ ....(4).

Extracting the square root, $ax^2-c=\pm(\sqrt{b^2-4ac})x$.................(5).

Adding equation (2), $2a^2x^2=(-b\pm\sqrt{b^2-4ac})x$..................(6).

Whence $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Let it be required to solve the equation $3x^2-2x=21$.

Transposing $2x$ to the second member and 21 to the first, the equation becomes

$3x^2-21=2x$.......................... .... (7).

Squaring, $9x^4-126x^2+441=4x^2$ ....(8).

Adding twice $126x^2$, $9x^4+126x^2+441=256x^2$ ....(9).

Extracting the square root, $3x^2+21=\pm16x$ ....(10).

Adding equation (7), $6x^2=18x$ or $-14x$.

$\therefore$ $x=3$ or $-2\frac{1}{3}$.
Is this new?

[NOTE.—We do not remember of ever having seen this method. If any of our readers have seen it elsewhere, please let us know. Editor.]

ON THE DOCTRINE OF PARALLELS.

By Hon. JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

I desire to enter my protest against any assumption that parallel lines, extended to an infinite distance, do, or do not, intersect. The human mind cannot comprehend the infinite and, therefore, we cannot determine the question. We may use modes of reasoning involving infinite quantities, but we can rely upon the results *only so far as human experience shows that they are correct.* It is trne, that a mode of reasoning in such cases. which leads to a result found by human experience to be correct in a particular case, may generally be assumed to be correct in all cases. Without human experience, the proposition that if two objects are moving in the same line in the same direction at different velocities, the one in advance will move over an appreciable space while the other is moving over the space between them and, therefore, that the one can never overtake the other, could never have been successfully denied. I hold that this doctrine applies to much of the discussion of the present day, and some of the propositions I have been able to deny, and old propositions denied I have been able to affirm, because I knew that *human experience had settled the matter.*

Whether Euclid's reasoning was, or was not correct, I have never seen a case in which the result which he reached has not been found to be absolutely correct by human experience.

The quotation which Professor Lyle makes from Lotze (Vol. II. page 375) involves the arrogant assumption that the human mind is infinite in the scope of its reasoning power. Mathematicians, of all men, should not claim that a proposition involving the infinite, cannot be true, because we cannot *comprehend the possibility* of its being true.